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New Junction Conditions for Signature Change: Null Boundary Proposal

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Existing literature on signature change considers the change surface to be space-like. We construct a new model for signature change based on the signature-changing hypersurface in the beginning of the universe being a null hypersurface. This affects most quantum cosmological considerations, since in quantum cosmology, according to the Hartle–Hawking no-boundary proposal, the creation of the Universe occurred along a space-like 3-hypersurface. Based on this assumption, junction conditions for the signature-change hypersurface are derived and the corresponding results are explained. The energy–momentum of the change surface is non-vanishing and the result for pressure of the change surface is in agreement with inflationary scenario.

KEY WORDS: signature change; null hypersurfaces; junction conditions.
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1. INTRODUCTION

In this section we give a short outline to the origin of the idea of signature change and some of its main aspects.

Hartle and Hawking (1983; Hawking, 1984), in constructing a satisfactory model of the Universe, try to avoid the initial spacetime singularity predicted by the standard model of cosmology, using a combination of general theory of relativity and quantum mechanics. The basic features of the so-called "Hawking Universe" obtained are as follows:

1. A satisfactory theory of quantum gravity will represent the gravitational field, in the manner of general theory of relativity, by a curved spacetime Hartle and Hawking (1983; Hawking, 1984).
2. The proper understanding of ordinary quantum mechanics is provided by Feynman's "path-integral" or "sum-over-histories" interpretation. In

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ordinary quantum mechanics the basic idea is that a quantum particle does not follow a single "path" between two spacetime points, and so does not have a single "history," but rather we must consider all possible "paths" connecting these points. Therefore the usual wave function Ψ is interpreted as an integral over all possible "paths" that a quantum system may take between two state. To solve the path integral, however, one must rotate the time variable in the usual quantum mechanical wave function to imaginary values in the complex plane, which yields as the new time coordinate the "Euclidean" time $\tau = it$ (Hartle and Hawking, 1983; Hawking, 1984; Halliwell and Hartle, 1990).

3. There is a wave function for the entire universe Ψ_U that is given by a Feynman path integral. The basic idea here is that one sums over all possible four-dimensional spacetimes (or spacetime "histories") connecting two three-dimensional spaces (states). In order to evaluate the path integral, however, one must again rotate the time variable to imaginary values which changes the integral from Lorentzian to Euclidean one. The result is that the temporal variable in the wave function is changed to a spatial one. In other words, Ψ_U sums only over Euclidean spacetimes, that is, over four-dimensional spaces with positive definite signature $(++++)$ (Hartle and Hawking, 1983; Hawking, 1984; Halliwell and Hartle, 1990; Gibbons and Hartle, 1990).
4. One wants to reach a certain state S at which the evolution of the universe becomes classical, in accordance with general theory of relativity and standard model of cosmology. Accordingly, Hawking proposes a path integral over the Euclidean four-space $g_{\mu\nu}$, and matter-field configurations ϕ that yields S. S is characterized by the three-metric h_{ij} and a value of the scalar field, ϕ.
5. To avoid an initial spacetime singularity, the cosmic path integral will include only compact (or closed) four-geometries, so that the three-geometries, marking successive states of the universe, shrink to zero in a smooth, regular way (Gibbons and Hawking, 1992). Hawking's universal wave function is obtained, therefore, by integrating only over compact four-geometries (Euclidean "spacetimes") that have the 3-space S as the only (lower) boundary and are such that a universe in state S will subsequently evolve.

Statements (3) and (5) above are the essence of the idea that the universe was initially Euclidean and then, by change of signature of "spacetime" metric, the transition to usual Lorentzian spacetime occurred. Earlier attempts to describe this interesting aspect was based on Euclidean path integral formulation of quantum gravity and the analogy to quantum tunnelling effect in quantum mechanics (Hartle and Hawking, 1983; Hawking, 1984; Halliwell and Hartle, 1990; Gibbons and Hartle, 1990; Gibbons and Hawking, 1992; Isham, 1989; Horowitz, 1991; Hawking, 1994).

The rise of this idea led many authors to consider it within the classical theory of general relativity (Sakharov, 1984; see also, Al'tshuler and Barvinsky, 1996; Ellis *et al.*, 1992; Ellis, 1992; Hayward, 1992, 1993, 1994; Dereli and Tucker, 1993; Kossowski and Kriele, 1993a,b; Hayward, 1995/1996; Hellaby and Dray, 1994; Kriele and Martin, 1995; Martin, 1994, 1995; Hayward, 1995a,b; Hellaby and Dray, 1995; Dray *et al.*, 1991, 1993; Dereli *et al.*, 1993; Egusquiza, 1995; Maia and Monte, 1995; Alty and Fewster, 1995; Dray *et al.*, 1995; Kriele, 1996; Dray, 1996; Iliev, 1998). The common feature of these attempts is that all of them consider the change surface as a space-like hypersurface. Ellis and his coworkers (Ellis *et al.*, 1992; Ellis, 1992), have shown that classical Einstein field equations, suitably interpreted, allow a change of signature of spacetime metric along a space-like boundary. They have also constructed specific examples of such changes in the case of Robertson–Walker geometries. Hayward (Hayward, 1992, 1993, 1994) gives the junction conditions necessary to match a region of Lorentzian-signature spacetime to a region of Euclidean-signature space across a space-like surface using vacuum Einstein or Einstein-Klein-Gordon equations.

Hellaby and Dray have shown that signature change leads to a finite source term for the signature changing surface (Hellaby and Dray, 1994). This should have led them to the result that the right-hand side of the Einstein equation should have a term proportional to Dirac δ function. Kriele and Martin (1995) do not accept the usual belief that signature change could be used to avoid space-time singularities, unless one abandon the Einstein equations at the signature changing surface. They also claim that there is no singularity at the signature changing surface due to concentration of matter. For more complete review see (Sakharov, 1984; Al'tshuler and Barvinsky, 1996; Ellis *et al.*, 1992; Ellis, 1992; Hayward, 1992, 1993, 1994; Dereli and Tucker, 1993; Kossowski and Kriele, 1993a,b; Hayward, 1995/1996; Hellaby and Dray, 1994; Kriele and Martin, 1995; Martin, 1994, 1995; Hayward, 1995a,b; Hellaby and Dray, 1995; Dray *et al.*, 1991, 1993; Dereli *et al.*, 1993; Egusquiza, 1995; Maia and Monte, 1995; Alty and Fewster, 1995; Dray *et al.*, 1995; Kriele, 1996; Dray, 1996; Iliev, 1998; Mansouri and Nozari, 2000; Nozari and Mansori, 2002). Recently Aguirre and Gratton, by constructing a null boundary proposal for inflation, have shown that if one consider cosmological boundary conditions on an infinite null boundary, there is no beginning of time and this is completely in accordance to our model (Aguirre and Gratton, 2003).

Here we want to argue that actually signature change hypersurface is a null hypersurface. Since the matching of the Schwarzschild geometry to FRW geometry along a common boundary is possible, we will conclude that actually the beginning of the Universe occur on a null hypersurface which in the case of general spherisymmetric geometry it leads to 2-spheres and these 2-spheres have the same rule as 3-spheres in "No Boundary" proposal. Since these 2-spheres are compact and nonsingular, therefore the problem of singularity completely resolves. We first construct a distributional approach for treating null hypersurface in general relativity (Poisson, 2002; Nozari and Mansouri, 2002) and then junction conditions for null signature change hypersurface are derived. The non-vanishing energy–momentum tensor of this hypersurface is in agreement with the work by Hellaby and Dray and also other literatures which accept the existence of non-vanishing energy–momentum tensor for signature change hypersurface (Mansouri and Nozari, 2000).

The organization of the paper is as follow: in Section 2 we construct a distributional formalism for treating null hypersurfaces in general relativity using admissible coordinates. Section 3 is devoted to our new formalism and proposal for signature change hypersurface and explaining the results. The paper follows by conclusions.

We use the signature $(-+++)$ for Lorentzian manifolds, and follow the curvature conventions of Misner, Thorne and Wheeler (Misner *et al.*, 1973). The square brackets, $[F]$, are used to indicate the jump of any quantity F at the layer, and the terms proportional to δ-function in equations, are denoted by $\check{F}$.

2. NULL-SHELL DISTRIBUTIONAL FORMALISM

Consider a space-time Manifold M consisting of two overlapping domains M_+ and M_- with metrics $g^+_{\alpha\beta}(x^\mu_+)$ and $g^-_{\alpha\beta}(x^\mu_-)$ in terms of independent disconnected charts x^μ_+ and x^μ_-, respectively. The common boundary of the domains is denoted by Σ, and taken to be lightlike. In other words, the manifolds M_+ and M_- are glued together along the null hypersurface, Σ. Introducing a single chart x^μ called "admissible coordinate system" which covers the overlap region and reaches into both domains, we write down the parametric equation of Σ as $\Phi(x^\mu) = 0$, where Φ is a smooth function. The domains of M in which Φ is positive or negative are contained in M_+ or M_-, respectively. By applying the coordinate transformations $x^\mu_\pm = x^\mu_\pm(x^\nu)$ on corresponding domains, a pair of metrics $g^+_{\alpha\beta}(x^\mu)$ and $g^-_{\alpha\beta}(x_\mu)$ is formed over M_+ and M_-, respectively, each suitably smooth (at least C^3).

There are two alternative approach to dynamics of hypersurfaces in general relativity: Darmois-Israel and distributional approach (Nozari and Mansori, 2002a). Here we give a new outline to distributional approach on the basis of our previous work (Nozari and Mansouri, 2002b) by keeping in mind that some corrections based on Poisson recent paper (Poisson, 2002) should be considered.

The main step in distributional approach is the definition of a hybrid metric $g_{\alpha\beta}(x^\mu)$ over M which glues the metrics $g^+_{\alpha\beta}(x^\mu)$ and $g^-_{\alpha\beta}(x_\mu)$ together continuously on Σ

$$g_{\alpha\beta} = g^+_{\alpha\beta}\theta(\Phi) + g^-_{\alpha\beta}\theta(-\Phi), \tag{1}$$

where θ is the Heaviside step function and

$$[g_{\alpha\beta}(x^{\mu})] = 0 \tag{2}$$

We expect on Σ the curvature and Ricci tensor to be proportional to δ function. It follows from (1) and (2) that the first derivative of $g_{\alpha\beta}$ is proportional to the step function. The δ distribution can only occur in the second derivative of the metric which enters linearly in the expressions for curvature and Ricci tensor. So the only relevant terms in the Ricci tensor are

$$\breve{R}_{\mu\nu} = \breve{\Gamma}^{\rho}_{\mu\rho,\nu} - \breve{\Gamma}^{\rho}_{\mu\nu,\rho}. \tag{3}$$

Using the metric in the form of (1), we finally arrive at the following expression for the components of the Ricci tensor proportional to δ distribution (Nozari and Mansouri, 2002a)

$$\breve{R}_{\mu\nu} = \left(\frac{1}{2g}[g_{,\mu}]\partial_{\nu}\Phi - \left[\Gamma^{\rho}_{\mu\nu}\right]\partial_{\rho}\Phi\right)\delta(\Phi(x)). \tag{4}$$

where g is the determinant of the metric and partial derivatives are done with respect to the admissible coordinates x^{μ}.

The intrinsic coordinates of Σ adapted to its null generators are taken to be $\xi = (\eta, \theta^{A})$, where η being an arbitrary parameter (not necessarily an affine parameter on both side of Σ) on the null generators of the hypersurface and θ^{A} are used to label the generators. Now we introduce tangent vectors $e^{\mu}_{a} = \frac{\partial x^{\mu}}{\partial \xi^{a}}$, naturally segregated into a null normal vector $n^{\mu} = \alpha^{-1}\partial_{\mu}\Phi$ that is also tangent to the generators, and two space-like vectors e^{μ}_{A} that point in the directions transverse to the generators

$$n^{\mu} = \left(\frac{\partial x^{\mu}}{\partial \eta}\right)_{\theta^{A}} \equiv e^{\mu}_{\eta}, \quad e^{\mu}_{A} = \left(\frac{\partial x^{\mu}}{\partial \xi^{A}}\right)_{\eta}. \tag{5}$$

One can show that $n^{\mu}n_{\mu} = n_{\mu}e^{\mu}_{A} = 0$. Now we should complete the partial basis e^{μ}_{a} by adding a transverse null vector N^{μ} with the following properties,

$$N^{\mu}N_{\mu} = 0, \quad N_{\mu}n^{\mu} = -1 \quad \text{and} \quad N_{\mu}e^{\mu}_{A} = 0. \tag{6}$$

The intrinsic metric on Σ is written as

$$\gamma_{AB} = g_{\mu\nu}e^{\mu}_{A}e^{\nu}_{B}, \tag{7}$$

which is the same on both sides of Σ. Expressed in terms of admissible coordinates, x^{μ}, on Σ we have,

$$[\gamma_{AB}] = [n^{\mu}] = [N^{\mu}] = \left[e^{\mu}_{A}\right] = [\alpha] = 0. \tag{8}$$

The energy–momentum tensor of the shell, $\check{T}_{\mu\nu}$, considered as a distribution is given by,

$$\check{T}_{\mu\nu} = |\alpha| S_{\mu\nu} \delta(\Phi) \tag{9}$$

where α is related to the transverse null vector N^{μ} of the shell as

$$\alpha = -N^{\mu} \partial_{\mu} \Phi, \tag{10}$$

and $S_{\mu\nu}$ is the surface tensor of energy–momentum of the shell expressed in admissible coordinates x^{μ} as follows (Poisson, 2002)

$$-\epsilon S^{\mu\nu} = \sigma n^{\mu} n^{\nu} + j^{A} \left(n^{\mu} e^{\nu}_{A} + e^{\mu}_{A} n_{\nu} \right) + p \gamma^{AB} e^{\mu}_{A} e^{\nu}_{B} \tag{11}$$

where $\epsilon = \frac{|\alpha|}{\alpha}$. The first term represents a flow of matter along the null generators of the hypersurface, and hence σ represents a mass density. The second term represents a flow of matter in the direction transverse to the generators, and j therefore represents a current density. The surface quantity p represents an isotropic pressure.

Now we can write Einstein's field equation for the light-like hypersurface Σ as:

$$\check{G}_{\mu\nu} = \kappa \check{T}_{\mu\nu}. \tag{12}$$

Defining,

$$\begin{aligned} Q_{\mu\nu} &= \alpha^{-1} \left(\frac{1}{2g} [g_{,\mu}] \delta^{\rho}_{\nu} - \left[\Gamma^{\rho}_{\mu\nu} \right] \right) \partial_{\rho} \Phi \\ &= \left(\frac{1}{2g} [g_{,\mu}] \, \delta^{\rho}_{\nu} - \left[\Gamma^{\rho}_{\mu\nu} \right] \right) n_{\rho} \end{aligned} \tag{13}$$

we obtain, using equations (23) and (33) for the energy–momentum tensor, the field equations in the four-dimensional form

$$Q_{\mu\nu} - \frac{1}{2} g_{\mu\nu} Q = \epsilon \kappa S_{\mu\nu} \tag{14}$$

where $Q = Q_{\mu\nu} g^{\mu\nu}$ and $\epsilon = \frac{|\alpha|}{\alpha}$. $Q_{\mu\nu}$ is a tensor with support on Σ and this equation obtained in admissible coordinate system, describes the dynamics of light-like hypersurface Σ in distributional approach.

3. NEW PROPOSAL: SIGNATURE CHANGE HYPERSURFACE IS A NULL HYPERSURFACE

In this section we want to construct a new formalism for signature change in early Universe based on this idea that signature changing hypersurface is a null hypersurface. Also junction conditions for signature change are derived in this situation. As we have indicated in introduction, Hartle and Hawking pioneer work and forthcoming papers by other authors consider this hypersurface as space-like hypersurface. So the assumption that this hypersurface being a null boundary, departs significantly from literatures and actually as we will argue, this assumption is more reasonable in some physical intuition.

For this purpose, we consider the simple case of spherical symmetry. Expressed in terms of Eddington retarded or advanced time u, the metric of a general spherisymmetric geometry is

$$ds^2 = -e^{\psi} du(fe^{\psi} du + 2\xi dr) + r^2\, d\Omega^2 \tag{15}$$

where $\psi_\pm$ and $f_\pm$ are two arbitrary functions of coordinates $u_\pm$ and $r_\pm$ in different side of Σ. The sign factor ξ is $+1$ if r increases toward the future along a ray $u = constant$, i.e., if the light cone $u = constant$ is expanding; if it contracts then $\xi = -1$. It is convenient to introduce a local mass function $m_\pm(u_\pm, r_\pm)$ defined as $f_\pm = 1 - \frac{2m_\pm}{r_\pm}$. We consider a thin shell whose history Σ, a light cone $u = constant$, splits spacetime into past and future domains M_- and M_+. We want to glue two spacetimes manifolds, M_- and M_+ along the hypersurface Σ using the results of the last section (the so-called distributional approach). Since we want to construct a model for signature change, we first set the metric (15) in the following form

$$ds^2 = -e^{\psi} du(fg(u)e^{\psi}\, du + 2\xi\, dr) + r^2\, d\Omega^2 \tag{16}$$

where $g(u)$ is defined as,

$$g(u) = \Theta(u) - \Theta(-u), \tag{17}$$

and Θ is Heaviside step function. We can do this in two different way. As a First approach we can write the Schwarzschild metric with an appropriate lapse function as in (15) and then perform Eddington–Finkelstein coordinates transformations. Second approach is that one can use general spherisymmetric geometry from beginning with metric (15) which has the lapse function as (13). Since these two approach are equivalent we use the second approach. Using the results of the last section, specially admissible coordinate system and definition of normal Gaussian coordinates with corresponding unit vectors, one can show that the non-vanishing components of $Q_{\mu\nu}$ from Eq. (14) are

$$Q_{uu} = \frac{\xi[fg]}{r} e^{2\psi_-} - \xi[\partial_r \psi] f_- g_- e^{2\psi_-}|_\Sigma \tag{18}$$

$$Q_{rr} = 4\xi \frac{[fg]}{rf_-^2 g_-^2}|_\Sigma \tag{19}$$

$$Q_{ur} = Q_{ru} = \frac{2[fg]}{rf_- g_-} e^{\psi_-} - [\partial_r \psi] e^{\psi_-}|_\Sigma \tag{20}$$

where $|_\Sigma$ means that relevant terms should be calculated on Σ. Also one can calculate Q as

$$Q = 2\xi[\partial_r \psi]|_\Sigma \tag{21}$$

and from (11) the nonzero components of the surface energy tensor, $S_{\mu\nu}$ are found as

$$S_{uu} = -\sigma e^{2\psi_-}|_\Sigma \tag{22}$$

$$S_{rr} = -4\sigma f_-^2 g_-^2|_\Sigma \tag{23}$$

$$S_{ur} = -2\xi \frac{\sigma}{r f_- g_-} e^{\psi_-}|_\Sigma \tag{24}$$

and

$$S_{\theta\theta} = -pr^2|_\Sigma \quad S_{\varphi\varphi} = \sin^2\theta S_{\theta\theta}|_\Sigma. \tag{25}$$

Here we consider the case of outgoing Eddington–Finkelstein coordinates with $\xi = +1$ and since $[g] = 2$ and $g_- = -1$ we find

$$S_{uu} = -\sigma e^{2\psi_-}|_\Sigma \tag{26}$$

$$S_{rr} = -4\sigma f_-^{-2}|_\Sigma \tag{27}$$

$$S_{ur} = 2\frac{\sigma}{r f_-} e^{\psi_-}|_\Sigma \tag{28}$$

and finally, we obtain the following junction conditions for signature changing hypersurface using (11),

$$\sigma = \frac{[m]}{4\pi r^2}|_\Sigma \tag{29}$$

$$p = -\frac{1}{8\pi}[\partial_r \psi]|_\Sigma. \tag{30}$$

These are novel results and are important for following reasons: firstly these results indicate that energy–momentum of the signature change hypersurface is not vanishing. This is very interesting result since must of the literatures which have been discussed in introduction, consider vanishing energy–momentum tensor for change surface. Secondly, Since signature change hypersurface is a null boundary, it means that the "time" has began along a null light cone and therefore there is no beginning of time and this is in complete accordance with (Aguirre and Gratton, 2003). As third result, we see that the pressure in Eq. (30) is negative. This is the equation of the state for inflation and since the phase of the universe after creation is inflationary, this result is completely acceptable and therefore Eq. (30) is a possible framework for treating inflationary models.

4. CONCLUSIONS

We have seen that existing literatures about signature change, consider the change surface as a space-like hypersurface. In this paper we investigated the possibility that signature changing hypersurface in the beginning of the universe being a null hypersurface. As we have discussed, this proposal will affect must of the quantum cosmological considerations, since in quantum cosmology creation of the universe was occurred along a space-like 3-hypersurface according to Hartle–Hawking no-boundary proposal. Our new proposal suggests that actually $2 + 2$ decomposition of the spacetime metric (Brady *et al.*, 1996) is more suitable for treating quantum cosmological problems relative to $3 + 1$ decomposition of ADM. Also our model suggests that one must re-formulate the path integral formalism of quantum gravity, since in usual formalism, successive hypersurfaces which their intrinsic metric appears in correlation amplitude, is considered as 3-space-like slices. Now, one must re-formulate this formalism with successive 2-space-like boundaries. As has been indicated in the introduction, to avoid an initial spacetime singularity the cosmic path integral will include only compact (or closed) four-geometries, so that the two-geometries, marking successive states of the universe, shrink to zero in a smooth, regular way. Hawking's universal wave function is obtained, therefore, by integrating only over compact four-geometries (Euclidean "spacetimes") that have the 2-space S as the only (lower) boundary and these are such that a universe in state S will subsequently evolve.

Our model also suggests that the boundary conditions for the Wheeler-De Witt equation now change. This model probably constructs a possible mechanism for treating the problem of time in quantum cosmology, since it seems that the problem of time in quantum cosmology originates in the nature of the $3 + 1$ decomposition of spacetime metric. We have derived junction conditions for signature change hypersurface and the corresponding results are explained. It is shown that energy–momentum of the change surface is non-zero, in disagreement with most existing literature except ref. (Hayward, 1992, 1993, 1994).

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